Indian Statistical Institute

Back Paper 2018-2019

Analysis II, B.Math First Year

Time : 3 Hours Date : 10.06.2019 Maximum Marks : 100 Instructor : Jaydeb Sarkar

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(i) Answer all questions. (ii) X and Y are metric spaces. (iii) $\|x\|^2 = \sum_{i=1}^{n} x_i^2$.

(1) (10 marks) If $A \subseteq X$ is bounded, then prove that $\overline{A}$ is also bounded.

(2) (15 marks) Let $f : [0, 1] \to \mathbb{R}$ be a continuous function. Prove that f is Riemann integrable on $[0, 1]$.

(3) (15 marks) Prove that the product of two Riemann integrable functions is itself Riemann integrable.

(4) (15 marks) Let $A = \{x \in \mathbb{R}^n : 2 < \|x\|^2 < 3\}$, and let

$$f(x) = \cos \frac{1}{\|x\|^2 - 1} \qquad (x \in \mathbb{R}^n, \|x\| \neq 1).$$

Prove that f is uniformly continuous in A.

(5) (15 marks) Let $f(x, y) = \dfrac{xy}{\sqrt{x^2 + y^2}}$ for all $(x, y) \neq (0, 0)$, and $f(0, 0) = 0$. Is f differentiable at $(0, 0)$? Justify your answer.

(6) (15 marks) True or false (with justification)? "If $f : X \to Y$ is a continuous bijective function, then f^{-1} is also continuous."

(7) (15 marks) Let U be an open and connected subset of $\mathbb{R}^n$, and let $f : U \to \mathbb{R}$ be a differentiable function. If

$$(\nabla f)(x) = 0,$$

for all $x \in U$, then prove that f is constant on U.